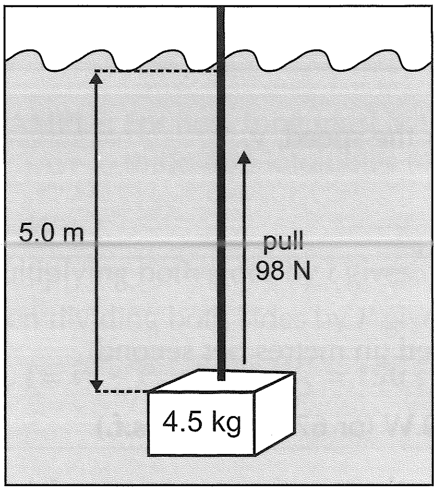
Efficiency

How Much of What You **Put In** Do You **Get Out**?

1) For most mechanical systems you **put in** energy in **one form** and the system **gives out** energy in **another**.
2) However, **some** energy is **always** converted into forms that **aren't useful**.
3) For example, an electric motor converts electrical energy into **heat** and **sound** as well as useful kinetic energy.
4) You can measure the **efficiency** of a system by the **percentage of total energy put in that is converted to useful forms.**

$$\textbf{Efficiency} = \frac{\textbf{Useful energy out}}{\textbf{Total energy in}} \times 100\%$$

EXAMPLE: A pirate uses a rope to pull a box of mass 4.5 kg vertically upwards through 5.0 m of water. He pulls with a force of 98 N. What is the efficiency of this system?



The **energy the pirate puts in** is the work he does pulling the rope.

The **useful energy out** is the gravitational potential energy gained by the box.

Some energy is converted to heat and sound by **friction** as the box is dragged through the water.

Total energy in = work done $= F \times s$
$= 98 \times 5.0$
$= 490$ J

Useful energy out = gravitational potential energy gained
$= m \times g \times h$
$= 4.5 \times 9.81 \times 5.0$
$= 220.725$ J

So, efficiency $= \frac{\text{Useful energy out}}{\text{Total energy in}} \times 100\%$

$= \frac{220.725}{490} \times 100\% = 45.045... =$ **45% (to 2 s.f.)**

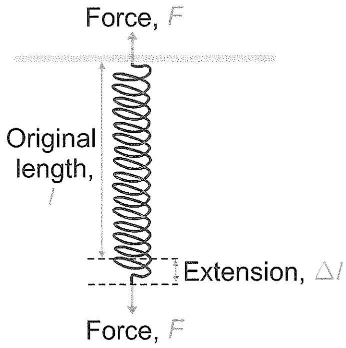
Efficiency – getting on with these questions instead of messing about...

1) A motor uses 375 joules of electrical energy in lifting a 12.9 kilogram mass through 2.50 metres. What is its efficiency?
2) It takes 1.4 megajoules (1.4×10^6 joules) of chemical energy from the petrol in a car engine to accelerate a 560 kilogram car from rest to 25 metres per second on a flat road.
   a) What is the gain in kinetic energy?
   b) What is the efficiency of the car?

Forces and Springs

Hooke's Law — Extension is Directly Proportional to Force

1) When you apply a **force** to an object you can cause it to **stretch** and **deform** (change shape).
2) **Elastic objects** are objects that return to their **original shape** after this deforming force is **removed**, e.g. springs.
3) When a **spring** is supported at the top and a **weight** is attached to the bottom, it **stretches**.
4) The **extension**, Δl, of a spring is **directly proportional** to the **force** applied, F. This is **Hooke's Law**.
5) This relationship is also true for many other elastic objects like **metal wires**.



force (in newtons, N) = **spring constant** (in newtons per metre, Nm^{-1}) × **extension** (in metres, m)

$$F = k \times \Delta l$$

The **spring constant**, k, depends on the stiffness of the **material** that you are stretching. It's measured in **newtons per metre** (Nm^{-1}).

EXAMPLE: A force is applied to a spring with a spring constant of 65.0 Nm^{-1}. The spring extends by 12.3 cm. What size is the force?

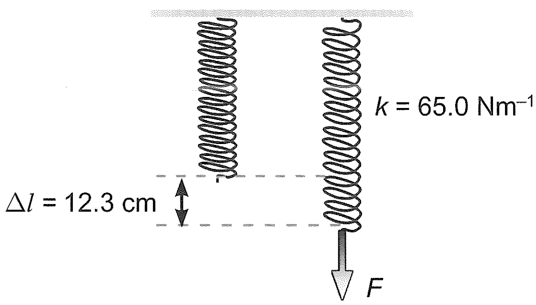
$F = k \times \Delta l$

$\Delta l = 12.3$ cm $= 0.123$ m

So, $F = 65.0 \times 0.123$

$= 7.995$

$=$ **8.00 N (to 3 s.f.)**



EXAMPLE: A sack of flour of mass 7.10 kg is attached to the bottom of a vertical spring. The spring constant is 85.0 Nm^{-1} and the top of the spring is supported. How much does the spring extend by?

$F = k \times \Delta l$, so $\Delta l = \frac{F}{k}$

You need to work out the force from the given mass:

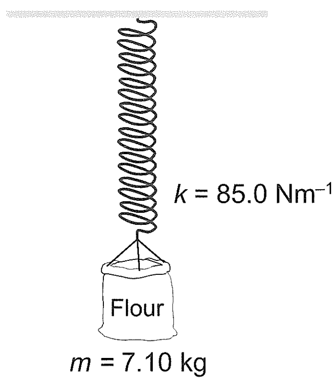
$F =$ weight of flour $= m \times g$

$= 7.10 \times 9.81 = 69.651$ N

So, $\Delta l = \frac{69.651}{85.0}$

$= 0.8194...$

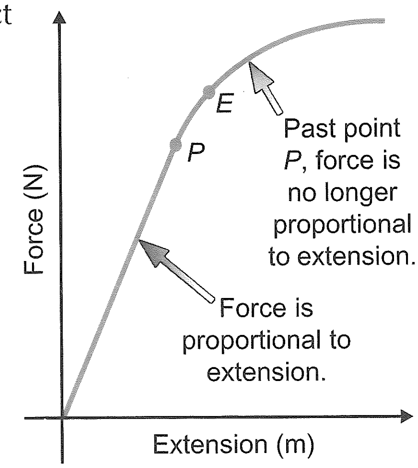
$=$ **0.819 m (to 3 s.f.)**



Forces and Springs

Hooke's Law **Stops Working** when the **Force** is **Great Enough**

There's a **limit** to the amount of force you can apply to an object for the extension to keep on increasing **proportionally**.



1) The graph shows **force** against **extension** for a spring.
2) For **small** forces, force and extension are **proportional**. So the first part of the graph shows a **straight-line relationship** between force and extension.
3) There is a **maximum force** that the spring can take and **still extend proportionally**. This is known as the **limit of proportionality** and is shown on the graph at the point marked ***P***.
4) The point marked ***E*** is the **elastic limit**. If you increase the force past this point, the spring will be **permanently stretched**. When the force is **removed**, the spring will be **longer** than at the start.
5) Beyond the **elastic limit**, we say that the spring deforms **plastically**.

Work Done can be **Stored** as **Elastic Strain Energy**

1) When a material is **stretched**, **work** has to be done in stretching the material.
2) If a deformation is **elastic**, all the work done is **stored** as **elastic strain energy** (also called **elastic potential energy**) in the material.
3) When the stretching force is removed, this **stored energy** is **transferred** to **other forms** — e.g. when an elastic band is stretched and then fired across a room, elastic strain energy is transferred to kinetic energy.
4) If a deformation is **plastic**, work is done to **separate atoms**, and energy is **not** stored as strain energy (it's mostly lost as heat).

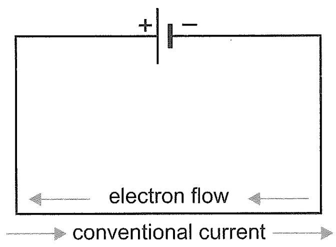
Spring into action — force yourself to learn all this...

1) A force applied to a spring with spring constant 64.1 Nm^{-1} causes it to extend by 24.5 cm. What was the force applied to the spring?
2) A pile of bricks is hung off a spring with spring constant 84.0 Nm^{-1}. The bricks apply a force of 378 N on the spring. How much does the spring extend by?
3) The mass limit for each bag taken on a flight with Cheapskate Airways is 9.0 kg. The mass of each bag is measured by attaching the bag to a spring.
   a) A bag of mass 7.4 kg extends the spring by 8.4 cm. What is the spring constant?
   b) The first bag is removed and another bag is attached to the spring. The spring extends by 9.5 cm. Can this bag be taken on the flight?
4) a) What is meant by the limit of proportionality?
   b) Why might a spring not return to its original length after having been stretched and then released?

Current and Potential Difference

Electric Current — the Rate of Flow of Charge Around a Circuit

1) In a circuit, **negatively-charged electrons** flow from the **negative** end of a battery to the **positive** end.
2) This flow of charge is called an **electric current**.
3) However, you can also think of current as a flow of **positive charge** in the **other direction**, from **positive** to **negative**. This is called **conventional current**.



The electric current at a point in the wire is defined as:

$$\textbf{current}\text{ (in amperes, A)} = \frac{\text{the \textbf{amount of charge} passing the point (in coulombs, C)}}{\text{the \textbf{time it takes} for the charge to pass (in seconds, s)}}$$

Or, in symbols: $I = \frac{Q}{t}$

> **EXAMPLE:** 585 C of charge passes a point in a circuit in 45.0 s. What is the current at this point?
>
> $I = \frac{Q}{t}$, so $I = \frac{585}{45.0}$ = **13.0 A**

Potential Difference (Voltage) — the Energy Per Unit Charge

1) In all circuits, energy is **transferred** from the power supply to the **components**.
2) The **power supply** does **work** on the **charged particles**, which **carry** this energy **around** the circuit.
3) The potential difference **across a component** is defined as the **work done** (or energy transferred) **per coulomb** of charge moved through the component.

$$\textbf{Potential difference across component}\text{ (in volts, V)} = \frac{\textbf{work done}\text{ (in joules, J)}}{\textbf{charge moved}\text{ (in coulombs, C)}}$$

In symbols: $V = \frac{W}{Q}$

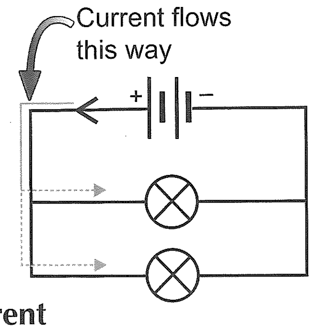
> **EXAMPLE:** A component does 10.8 J of work for every 2.70 C that passes through it. What is the potential difference across the component?
>
> $V = \frac{W}{Q}$, so $V = \frac{10.8}{2.70}$ = **4.00 V**

Physicists love camping trips — they get to study po-tent-ial difference...

1) How long does it take to transfer 12 C of charge if the average current is 3.0 A?
2) The potential difference across a bulb is 1.5 V.
How much work is done to pass 9.2 C through the bulb?
3) A motor runs for 275 seconds and does 9540 J of work.
If the current in the circuit is 3.80 A, what is the potential difference across the motor?

Current in Electric Circuits

Charge is **Always Conserved** in Circuits

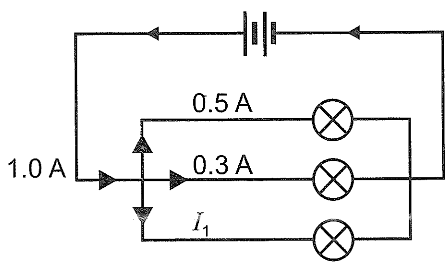


1) As **charge flows** through a circuit, it **doesn't** get **used up** or **lost**.
2) You can easily build a circuit in which the electric current can be **split** between **two wires** — two lamps connected in **parallel** is a good example.
3) Because charge is **conserved** in circuits, whatever charge flows **into** a junction will flow **out** again.
4) Since **current** is **rate of flow of charge**, it follows that whatever **current flows into** a junction is the **same** as the **current flowing out** of it.

   the **sum** of the **currents going into the junction** = the **sum** of the **currents going out**

   This is **Kirchhoff's first law**. It means that the current is the **same** everywhere in a **series circuit**, and is **shared between the branches** of a **parallel circuit**.
5) N.B. — current arrows on circuit diagrams normally show the direction of flow of **conventional current** (see p.25).

EXAMPLE: Use Kirchoff's first law to find the unknown current I_1.



Sum of currents in = sum of currents out

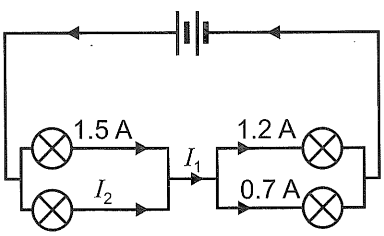
$1.0 = 0.5 + 0.3 + I_1$

$1.0 = 0.8 + I_1$

$I_1 = 1.0 - 0.8$

$I_1 =$ **0.2 A**

EXAMPLE: Calculate the missing currents, I_1 and I_2, in this circuit.



Looking at the junction immediately after I_1:

$I_1 = 1.2 + 0.7$

$I_1 =$ **1.9 A**

And looking at the junction immediately before I_1:

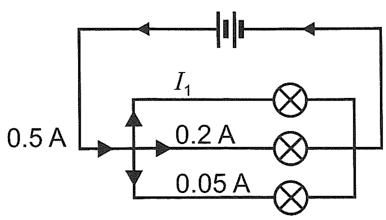
$1.5 + I_2 = 1.9$

$I_2 = 1.9 - 1.5$

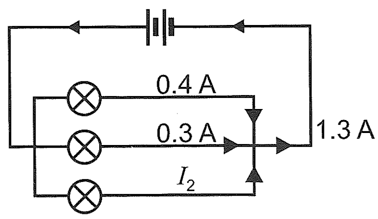
$I_2 =$ **0.4 A**

Conserve charge — make nature reserves for circuit boards...

1) What is the value of I_1?



2) What is the value of I_2?



Potential Difference in Electric Circuits

*Energy is **Always Conserved** in Circuits*

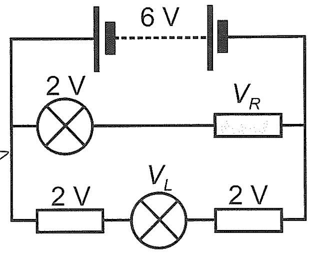
1) Energy is **given** to **charged particles** by the **power supply** and **taken off them** by the **components** in the circuit.
2) Since energy is **conserved**, the **amount** of energy one coulomb of charge loses when going around the circuit must be **equal to** the energy it's **given** by the power supply.
3) This must be true **regardless** of the **route** the charge takes around the circuit. This means that:

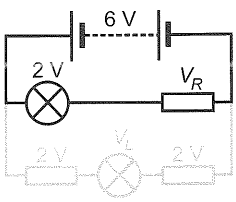
For any **closed loop** in a circuit, the **sum** of the **potential differences** across the components **equals** the **potential difference** of the **power supply**.

This **Kirchhoff's second law.** It means that:

- In a **series circuit**, the potential difference of the power supply is split between all the components.
- In a **parallel circuit**, each **loop** has the same potential difference as the power supply.

EXAMPLE: Use Kirchoff's second law to calculate the potential differences across the resistor, V_R, and the lamp, V_L, in the circuit shown on the right.



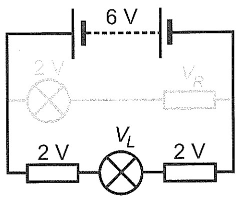


First look at just the top loop:

p.d. of power supply = sum of p.d.s of components in top loop

$6 = 2 + V_R$

So $V_R = 6 - 2 =$ **4 V**



Now look at just the outside loop:

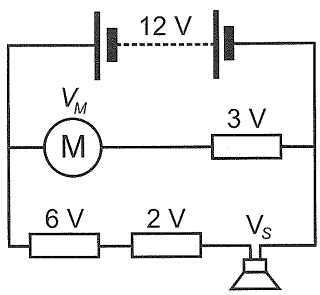
p.d. of power supply = sum of p.d.s of components in outside loop

$6 = 2 + V_L + 2$

So $V_L = 6 - 2 - 2 =$ **2 V**

This page is potentially tricky — so have a read of it all again...

1) For the circuit on the right, calculate:
   a) the voltage across the motor, V_M.
   b) the voltage across the loudspeaker, V_S.
2) A third loop containing two filament lamps is added to the circuit in parallel with the first two loops. What is the sum of the voltages of the two filament lamps?



Resistance

Resistance — The Ratio of Potential Difference to Current

1) If there's a potential difference **across** a component a **current** will **flow through it**.
2) Usually, as the **potential difference** is **increased** the **current increases** — this makes sense if you think of the potential difference as a kind of force **pushing** the charged particles.
3) You can link current and potential difference by defining "**resistance**":

$$\textbf{Resistance of component} \text{ (in ohms, } \Omega) = \frac{\textbf{potential difference across component} \text{ (in volts, V)}}{\textbf{current passing through component} \text{ (in amps, A)}}$$

Or, in symbols: $\boldsymbol{R = \frac{V}{I}}$

Multiplying both sides by I gives: $\boldsymbol{V = I \times R}$

4) Components with a **low resistance** allow a **large** current to flow through them, while components with a **high resistance** allow only a **small** current.
5) The resistance **isn't** always **constant** though — it can take **different values** as the **current** and **voltage change**, or it can change with conditions like **temperature** and **light level**.

EXAMPLE: If a potential difference of 12 V across a component causes a current of 1.0 mA to flow through it, what is the resistance of the component?

$R = \frac{V}{I}$, so $R = \frac{12}{1.0 \times 10^{-3}}$ = **12 000 Ω**, or 12 kΩ

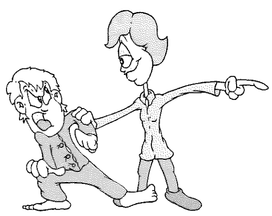
EXAMPLE: What potential difference must be applied across a lamp with a resistance of 200 Ω in order for a current of 0.2 A to flow through it?

$V = I \times R$, so $V = 0.2 \times 200$ = **40 V**

EXAMPLE: What current will flow through an 850 Ω resistor if a potential difference of 6.3 V is applied across it?

$V = I \times R$. Dividing both sides by R gives $I = \frac{V}{R}$,

so $I = \frac{6.3}{850}$ = 0.007411... = **0.0074 A** (or 7.4 mA) **(to 2 s.f.)**



Ohm my, look at that — more questions to do...

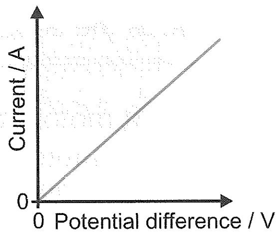
1) If a current of 2.5 amps flows through a component with a resistance of 15 ohms, what is the potential difference across the component?
2) What current will flow through a 2500 Ω resistor if the voltage across it is 6.0 volts?
3) What is the resistance of a component if 1.5 volts drives a current of 0.024 amps through it?

I-V Graphs

Ohm's Law Says *Potential Difference* is *Proportional* to *Current*

1) An *I-V* graph is a graph of **current** against **potential difference** for a component. For any *I-V* graph, the **resistance** at a given point is the potential difference divided by the current ($R = \frac{V}{I}$).
2) Provided the **temperature** is **constant**, the **current** through an **ohmic component** (e.g. a resistor) is **directly proportional** to the **potential difference** across it ($V \propto I$). This is called **Ohm's Law**.
3) An **ohmic component's** *I-V* graph is a **straight line**, with a gradient equal to 1 ÷ the resistance of the component. The **resistance** (and therefore the **gradient**) is **constant**.

- So for an ohmic component, **doubling** the **potential difference doubles** the **current**.
- Often **external factors**, such as **temperature**, will have a **significant effect** on resistance, so you need to remember that Ohm's law is **only** true for components like resistors at **constant temperature**.

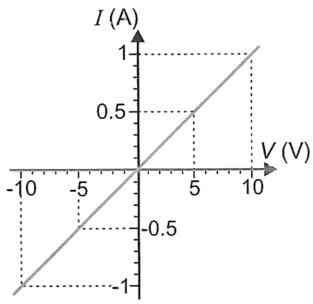


4) Sometimes you'll see a graph with **negative** values for p.d. and current. This just means the current is flowing the **other way** (so the terminals of the power supply have been switched).

EXAMPLE: Look at the *I-V* graph for a resistor on the right. What is its resistance when the potential difference across it is: a) 10 V, b) 5 V, c) –5 V, d) –10 V?

a) $R = \frac{V}{I} = \frac{10}{1} = 10\ \Omega$ b) $R = \frac{V}{I} = \frac{5}{0.5} = 10\ \Omega$

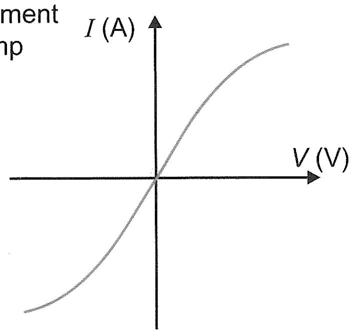
c) $R = \frac{V}{I} = \frac{-5}{-0.5} = 10\ \Omega$ d) $R = \frac{V}{I} = \frac{-10}{-1} = 10\ \Omega$

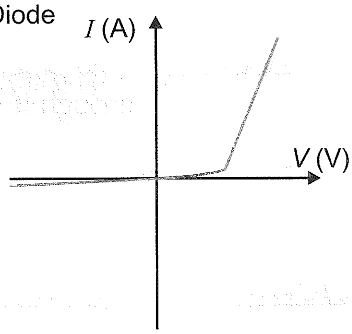


*I-V Graphs for **Other** Components **Aren't Straight Lines***

The *I-V* graphs for **other** components **don't** have **constant gradients**.
This means the resistance **changes** with voltage.

1) As the p.d. across a filament lamp gets **larger**, the filament gets **hotter** and its resistance **increases**.
2) Diodes only let current flow in **one direction**. The resistance of a diode is **very high** in the **other** direction.





I-Ve decided you need amp-le practice to keep your knowledge current...

1) State Ohm's law.
2) Sketch *I-V* graphs for: a) an ohmic resistor, b) a filament lamp, c) a diode.

Power in Circuits

Power — the Rate of Transfer of Energy

1) Components in electrical circuits transfer the **energy** carried by electrons into other forms.
2) The **work done each second** (or the **energy transferred each second**) is the **power** of a component:

$$\textbf{power} \text{ (in watts, W)} = \frac{\textbf{work done} \text{ (in joules, J)}}{\textbf{time taken} \text{ (in seconds, s)}}$$

Or, in symbols: $P = \frac{W}{t}$

This is the same as the equation for mechanical power that you saw on page 20.

> **EXAMPLE:** A lift motor does 3.0×10^5 J of work in a single one-minute journey. At what power is it working?
>
> $P = \frac{W}{t}$, so $P = \frac{3.0 \times 10^5}{60}$ = **5000 W** (or 5 kW)

Calculating Power from Current and Potential Difference

The work done is equal to the potential difference across the component multiplied by the amount of charge that has flowed through it (**$W = V \times Q$**) — see p.25.

So: $P = \frac{V \times Q}{t}$

The amount of charge that flows through a component is equal to the current through it multiplied by the time taken (**$Q = I \times t$**) — see p.25 again.

So: $P = \frac{V \times I \times t}{t}$

Cancelling the t's gives: $P = V \times I$

power (in watts) = **potential difference** (in volts) × **current** (in amps)

> **EXAMPLE:** If the potential difference across a component is 6 volts and the current through it is 0.50 milliamps (5.0×10^{-4} amps), at what rate is it doing work?
>
> $P = V \times I$, so $P = 6 \times 5.0 \times 10^{-4}$ = **0.003 W** (or 3 mW)

Knowledge is power — make sure you know these power equations...

1) What is the power output of a component if the current through it is 0.12 amps when the potential difference across it is 6.5 volts?
2) An electric heater has an operating power of 45 W.
   a) What current passes through the heater when the potential difference across it is 14 volts?
   b) How much work does the heater do in 12 seconds?

Power in Circuits

You Can Combine the Equations for Power and Resistance

You can **combine** the last equation for the power of an electrical component, $P = V \times I$, with the **equation** for resistance, $R = \frac{V}{I}$ (see p.28), to create two **more useful** equations.

1) Substitute $V = I \times R$ into $P = V \times I$ to get: $P = I \times R \times I = I^2R$

power (in watts) = [**current** (in amps)]^2 × **resistance** (in ohms)

2) Or substitute $I = \frac{V}{R}$ into $P = V \times I$ to get: $P = V \times \frac{V}{R} = \frac{V^2}{R}$

$$\text{power (in watts)} = \frac{[\text{potential difference (in volts)}]^2}{\text{resistance (in ohms)}}$$

Here are some examples — the key here is choosing the **right equation** to use. If the question gives you the value of two variables and asks you to find a third, you should choose the equation that relates these three variables. You might have to **rearrange** it before using it.

EXAMPLE: What is the power output of a component with resistance 100 Ω if the current through it is 0.2 A?

$P = I^2R$, so $P = 0.2^2 \times 100 =$ **4 W**

EXAMPLE: Resistors get hotter when a current flows through them. If you double the current through a resistor, what happens to the amount of heat energy produced every second?

It **increases by a factor of 4** — this is because the current is squared in the expression for the power (you can substitute some values of I and R in to check this).

EXAMPLE: If a lamp has an operating power of 6.5 W and the potential difference across it is 12 V, what is its resistance?

$P = \frac{V^2}{R}$, so multiplying both sides by R gives $P \times R = V^2$, and dividing by P gives:

$R = \frac{V^2}{P}$, so $R = \frac{12^2}{6.5} = 22.153... =$ **22 Ω (to 2 s.f.)**

(This answer is rounded to 2 s.f. to match the data in the question — see page 1.)

Watts up with your watch, Dr Watson? Dunno, but it sure is i²rksome...

1) What is the power output of a 2400 Ω component if the current through it is 1.2 A?
2) A motor has a resistance of 100 Ω. How much work does it do in 1 minute if it is connected to a 6 V power supply?
3) The current through a 6.0 W lamp is 0.50 A. What is the resistance of the lamp?