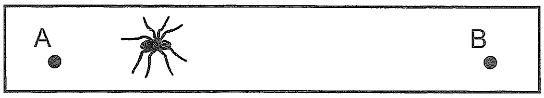
Speed, Displacement and Velocity

Distance, Time and Speed are all Related

Points A and B are separated by a **distance** in **metres**. Now imagine a spider walking from A to B — you can measure the **time** it takes, in **seconds**, for it to travel this distance.

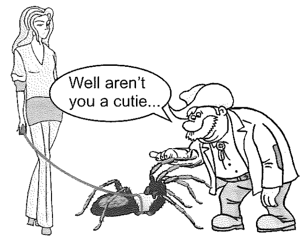


You can then work out the spider's **average speed** between A and B using this **equation**:

speed (in metres per second) = **distance travelled** (in metres) ÷ **time taken** (in seconds)

This is a very useful equation, but it does have a couple of **limitations**:

1) It only tells you the **average** speed. The spider could **vary** its speed from fast to slow and even go **backwards**. So long as it gets from A to B in the **same time** you get the **same answer**.
2) We assume that the spider takes the **shortest possible path** between the two points (a straight line), rather than **meandering** around.



*Displacement is a **Vector Quantity***

To get from point A to point B you need to know what **direction** to travel in — just knowing the **distance** you need to travel **isn't enough**.

This information, **distance plus direction**, is known as the **displacement** from A to B and has the symbol s. It's a **vector** quantity — **all** vector quantities have both a **size** and a **direction**.

*There is a **Relationship** Between **Displacement** and **Velocity***

Velocity is another **vector quantity** — velocity is the **speed** and **direction** of an object.

The **velocity** of an object is given by the following equation:

velocity (in metres per second) = **displacement** (in metres) ÷ **time taken** (in seconds)

Or, in symbols: $v = \frac{s}{t}$

This equation is very similar to the one relating **speed** and **distance**, except that it includes information about the **direction of motion**.

Displacement's in a relationship with velocity now, it's so over time...

1) An athlete runs a 1500 m race in a time of 210 seconds. What is his average speed?
2) The speed of light is 3.0×10^8 ms^{-1}. If it takes light from the Sun 8.3 minutes to reach us, what is the distance from the Earth to the Sun?
3) A snail crawls at a speed of 0.24 centimetres per second. How long does it take the snail to travel 1.5 metres?
4) How long does it take a train travelling with a velocity of 50 ms^{-1} north to travel 1 km?
5) If someone has a velocity of 7.50 ms^{-1} south, what is their displacement after 15.0 seconds?

Drawing Displacements and Velocities

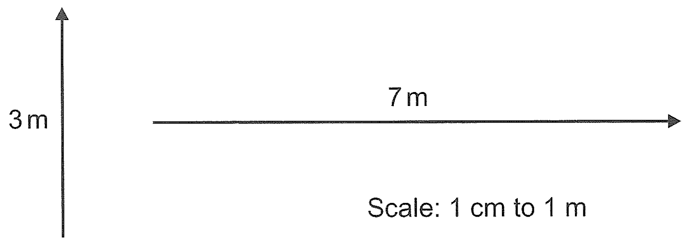
*You can use **Scale Drawings** to **Represent Displacement***

The simplest way to draw a vector is to draw an **arrow**. So for a displacement vector the **length** of the arrow tells you the **distance**, and the way the arrow **points** shows you the **direction**.

You can do this even for very large displacements so long as you **scale down**. Whenever you do a scale drawing, make sure you **state the scale** you are using.

EXAMPLE: Draw arrows to scale to represent a displacement of 3 metres upwards and a displacement of 7 metres to the right.

A displacement of 3 metres upwards could be represented by an arrow of length 3 centimetres. Using this same scale (1 cm to 1 m) a displacement of 7 metres to the right would be an arrow of length 7 centimetres.

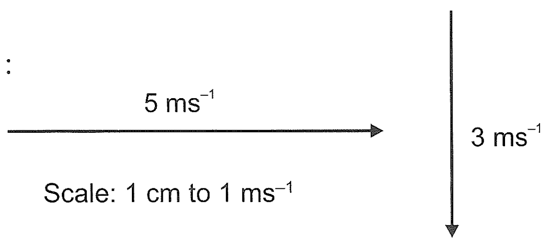


*You can also **Represent Velocities** with **Arrows***

Velocity is a **vector**, so you can **draw arrows** to show velocities too. This time, the **longer** the **arrow**, the **greater** the **speed** of the object. A typical scale might be 1 cm to 1 ms^{-1}.

EXAMPLE: Draw arrows to scale to represent velocities of 5 metres per second to the right and 3 metres per second downwards.

Draw the velocities like this with a scale of 1 cm to 1 ms^{-1}:



Drawing displacements — not about leaving your sketchbook at home...

1) Draw arrows representing the following displacements to the given scale:
   a) 12 m to the right (1 cm to 2 m)
   b) 110 miles at a bearing of 270° (1 cm to 20 miles)
2) Draw an arrow to represent each velocity to the given scale. Take north to be up the page.
   a) 60 ms^{-1} to the south-east (1 cm to 15 ms^{-1})
   b) 120 miles per hour to the west (1 cm to 30 miles per hour)

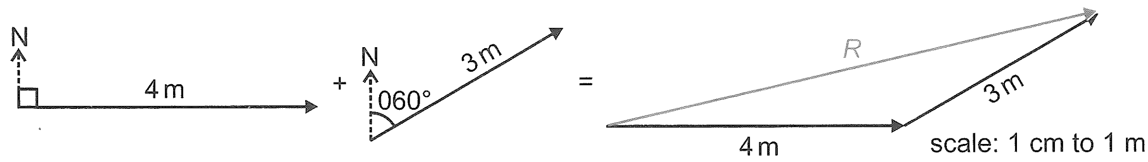
Combining Displacements and Velocities

You can use **Arrows** to **Add** or **Subtract Two Vectors...**

To **add** two velocity or displacement vectors, you **can't** simply add together the two distances as this doesn't account for the **different directions** of the vectors. What you do is:

1) **Draw** arrows representing the two vectors.
2) **Place** the arrows **one after the other** "tip-to-tail".
3) Draw a **third** arrow from start to finish. This is your **resultant vector**.

EXAMPLE: Add a displacement of 4 metres on a bearing of 090° to a displacement of 3 metres on a bearing of 060°. Use a scale of 1 cm to 1 m.



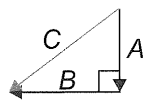
R is the **resultant** vector— it's the sum of the two displacements.
You can find the size of *R* by measuring the arrow and scaling up.
In this case it's 6.7 cm long which means the displacement is **6.7 m**.

To **subtract** vectors you need to **flip the direction** of the vector you are subtracting.
This **changes the sign** of the vector.
Adding the flipped vector is the **same** as **subtracting** the vector.

For example: 3 m → − 4 m → = 3 m → + ← (− 4 m) = ← −1 m

...Or Use **Pythagoras** if the Vectors make a **Right Angle Triangle**

If two vectors, *A* and *B*, are at right angles to each other, you can also use Pythagoras' theorem to find the resultant. ⇨ $A^2 + B^2 = C^2$

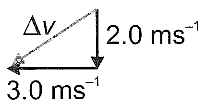


EXAMPLE: An object has an initial velocity of 3.0 ms^{-1} to the right, and a final velocity of 2.0 ms^{-1} down. Find the size of the change in velocity.

Change in velocity = Δv = final velocity – initial velocity.

First, flip the direction and change the sign of the vector that is being subtracted.

↓ 2.0 ms^{-1} − 3.0 ms^{-1} → = ↓ 2.0 ms^{-1} + ← 3.0 ms^{-1} =



$A^2 + B^2 = C^2$, so $C = \sqrt{A^2 + B^2} = \sqrt{2.0^2 + 3.0^2} = 3.605... =$ **3.6 ms^{-1} (to 2 s.f.)**
(This answer is rounded to 2 s.f. to match the data in the question — see page 1.)

Subtracting velocity vectors is easy — subtracting velociraptors, less so...

1) Find the size of the resultant of the following displacements by drawing the arrows "tip-to-tail".
   a) 5.0 m right and 4.0 m up.
   b) 15.0 miles south and 15.0 miles on a bearing of 045°.
2) Initial velocity = 1.0 ms^{-1} west and final velocity = 3.0 ms^{-1} north. Find the size of Δv.

Resolving Vectors

You can Split a Vector into Horizontal and Vertical Components

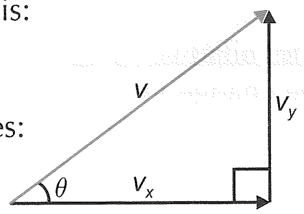
1) Vectors like **velocity** and **displacement** can be **split** into **components**.
2) This is basically the opposite of finding the resultant — you start from the resultant vector and split it into two separate vectors at **right angles** to each other.
3) Together these two components have the **same effect** as the **original** vector.
4) To find the components of a vector, v, you need to use **trigonometry**:

You get the **horizontal** component v_x like this:

$$\cos\theta = \frac{v_x}{v}$$

Rearranging this gives:

$$v_x = v\cos\theta$$



...and the **vertical** component v_y like this:

$$\sin\theta = \frac{v_y}{v}$$

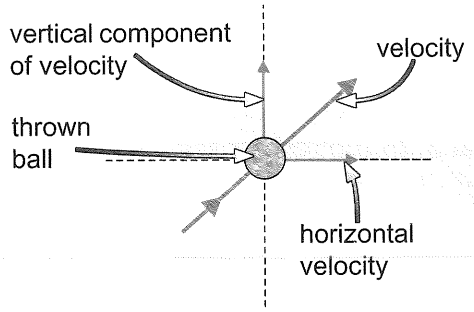
Rearranging this gives:

$$v_y = v\sin\theta$$

You can also **rearrange** these equations to find θ. E.g. if you know v_x and v then:

$$\theta = \cos^{-1}\left(\frac{v_x}{v}\right)$$

Resolving is dead useful because the two components of a vector **don't affect each other**. This means you can deal with the two directions **completely separately**.

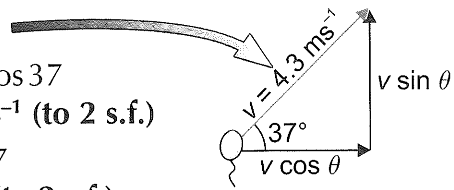


If you throw a ball diagonally up and to the right...

- Only the vertical component of the velocity is affected by gravity (see page 7).
- You can calculate the ball's vertical velocity (which will be affected by gravity).
- And you can calculate the ball's horizontal velocity (which won't be affected by gravity).

EXAMPLE: A helium balloon is floating away on the wind. It is travelling at 4.3 ms^{-1} at an angle of 37° to the horizontal. What are the vertical and horizontal components of its velocity?

It's useful to start off by drawing a diagram:



Horizontal velocity = $v_x = v\cos\theta = 4.3 \times \cos 37$
$= 3.434... =$ **3.4 ms^{-1} (to 2 s.f.)**

Vertical velocity = $v_y = v\sin\theta = 4.3 \times \sin 37$
$= 2.587... =$ **2.6 ms^{-1} (to 2 s.f.)**

Solve these questions by re-solving the vectors...

1) A rugby ball is moving at 12 ms^{-1} at an angle of 68° to the horizontal. Find the horizontal and vertical components of the ball's velocity.
2) A plane is travelling at 98 ms^{-1} at a constant angle as it gains altitude. The horizontal velocity of the plane is 67 ms^{-1}. What is its angle of ascent?
3) A hot air balloon descends at a velocity of 5.9 ms^{-1} at an angle of 23° to the horizontal. How long does it take the balloon to descend 150 m?

Acceleration

Acceleration — *the Change in Velocity Every Second*

Acceleration is the **rate of change** of **velocity**. Like velocity, it is a **vector quantity** (it has a size and a direction). It is measured in **metres per second squared** (ms^{-2}).

$$\textbf{Acceleration} \text{ (in metres per second}^2\text{)} = \frac{\textbf{change in velocity} \text{ (in metres per second)}}{\textbf{time taken} \text{ (in seconds)}}$$

So: $$\textbf{Acceleration} = \frac{\textbf{final velocity} - \textbf{initial velocity}}{\textbf{time taken}}$$

Or in symbols: $a = \frac{v - u}{t} = \frac{\Delta v}{t}$ where u is the initial velocity, v is the final velocity and Δv is the change in velocity.

You'll often only need to think about velocities in **one dimension**, say left to right.

But you still need to recognise the **difference** between velocities from right to left and velocities from left to right.

Choose a direction to be **positive** — below, we'll use **right**. All velocities in this direction will from now on be positive, and all those in the **opposite direction** (left) will be **negative**.

Deceleration is negative acceleration and acts in the **opposite direction** to motion.

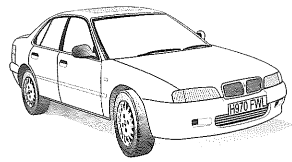
EXAMPLE: A car starts off moving to the right at 15.0 metres per second. After 30.0 seconds it is moving to the left at 5.25 metres per second. What was its acceleration during this time?

$u = 15.0\ ms^{-1}$ to the right $= +15.0\ ms^{-1}$

$v = 5.25\ ms^{-1}$ to the left $= -5.25\ ms^{-1}$

So, $a = \frac{v - u}{t} = \frac{-5.25 - 15.0}{30.0} = \frac{-20.25}{30.0} = \mathbf{-0.675\ ms^{-2}}$

(The acceleration is negative so it's to the left.)



EXAMPLE: A dinosaur accelerates from rest at $4.00\ ms^{-2}$ to the right. If its final velocity is $25.0\ ms^{-1}$ to the right, how long does it accelerate for?

$u = 0.00\ ms^{-1}$ $v = 25.0\ ms^{-1}$ to the right $= +25.0\ ms^{-1}$

$a = \frac{v - u}{t}$, multiplying both sides by t gives $a \times t = v - u$,

and then dividing both sides by a gives $t = \frac{v - u}{a}$. So, $t = \frac{25.0 - 0}{4.00} = \mathbf{6.25\ s}$

A seller rating is the key thing to check when buying a car online...

1) A train has an initial velocity of $12.8\ ms^{-1}$ to the left. After 22.0 seconds it is moving to the right at $18.3\ ms^{-1}$. What was its average acceleration during this time?
2) A ship accelerates at a uniform rate of $0.18\ ms^{-2}$ east. If its initial velocity is $1.5\ ms^{-1}$ east and its final velocity is $4.5\ ms^{-1}$ in the same direction, how long has it been accelerating for?
3) A rabbit is hopping at a constant speed when he begins decelerating at a rate of $0.41\ ms^{-2}$. What was the rabbit's initial hopping speed if it takes him 3.7 seconds to come to a stop?

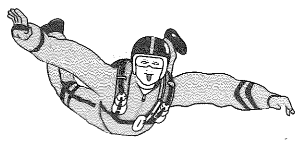
Acceleration Due To Gravity

The Acceleration Due to Gravity is g

When an object is dropped, it accelerates downwards at a constant rate of roughly $9.81\ ms^{-2}$. This is the **acceleration due to gravity** and it has the symbol ***g***.

It usually seems sensible to take the upward direction as positive and down as negative, making the acceleration due to gravity $\mathbf{-9.81\ ms^{-2}}$.

EXAMPLE: What is the vertical velocity of a skydiver 5.25 seconds after she jumps out of a plane that is travelling at a constant altitude? (Ignore air resistance and horizontal motion.)



$u = 0$

$a = -9.81\ ms^{-2}$

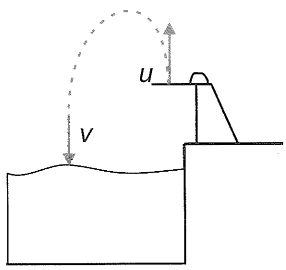
You can rearrange $a = \frac{v-u}{t}$ to give $v = u + (a \times t)$.

So $v = 0 + (-9.81 \times 5.25)$

$= 0 - 51.5025$

$= -51.5025 =$ **51.5 ms⁻¹ down (to 3 s.f.)**

EXAMPLE: A diver jumps up off a springboard. After 2.50 seconds he hits the water travelling downwards at $18.0\ ms^{-1}$. What was his initial vertical velocity? (Ignore air resistance and horizontal motion.)



$v = 18.0\ ms^{-1}$ down $= -18.0\ ms^{-1}$

$a = -9.81\ ms^{-2}$

You can rearrange $a = \frac{v-u}{t}$ to give $u = v - (a \times t)$.

So, $u = -18.0 - (-9.81 \times 2.50)$

$= -18.0 - (-24.525)$

$= -18.0 + 24.525$

$= 6.525 =$ **6.53 ms^{-1} upwards (to 3 s.f.)**

This isn't falling, it's learning with style...

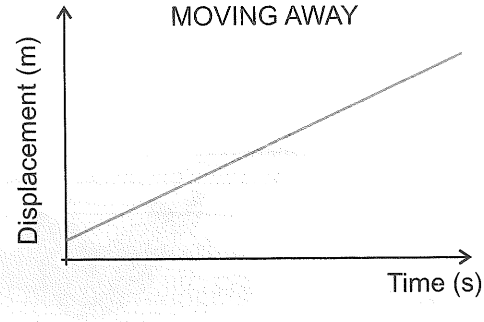
You can ignore air resistance in these questions. Hint — drawing a little diagram can help.

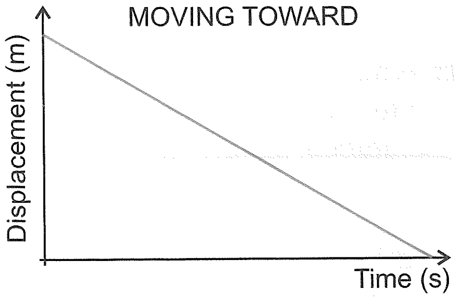
1) An apple falls from a tree and hits the ground at $4.9\ ms^{-1}$. For how long was it falling?
2) A stone is thrown straight downwards. It hits the ground at $26.5\ ms^{-1}$ after 2.15 seconds. What velocity was it thrown at?
3) A metal rod falls from a stationary helicopter. What velocity does it hit the ground at, 10.0 seconds later?
4) A sandbag is dropped from a stationary hot-air balloon. It hits the ground at a velocity of 24.5 metres per second. How long was it falling for?
5) A ball is thrown straight upwards. After 1.90 seconds it is moving downwards at $10.7\ ms^{-1}$ and is caught. With what velocity was it thrown?

Displacement-Time Graphs

You can **Draw Graphs** to Show **How Far** Something has **Travelled**

1) A graph of displacement against time tells you **how far** an object is from a given point, in a given direction, as time goes on.
2) As the object moves **away** from that point the **displacement** on the graph goes **up**, and as it moves **towards** it the displacement goes **down**:

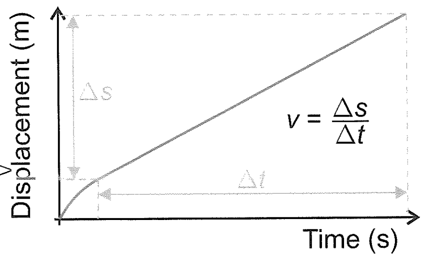




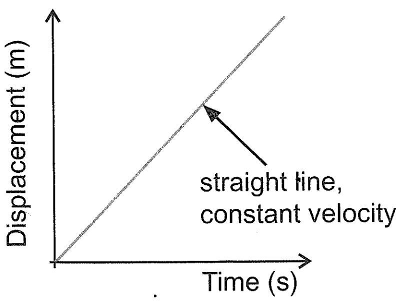
3) Important — these graphs only tell you about motion in **one dimension**. For example, a graph could tell you **how far up** a ball has been thrown, but **not** how far it has **moved horizontally**.

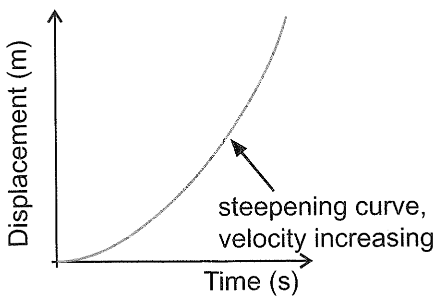
The **Gradient** of the **Line** is the **Velocity**

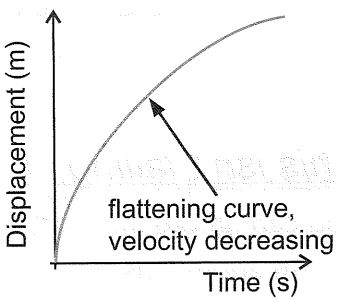
Velocity = displacement ÷ time (see p.2), so the **gradient** (slope) of a displacement-time graph tells you **how fast** an object is travelling, and **what direction** it is moving in. The **greater** the **gradient**, the **larger** the **velocity**.



1) If the line is **straight**, the velocity is **constant**.
2) If the line is **curved**, the velocity is **changing** — the object is **accelerating** or decelerating.
3) A **steepening curve** means the **object** is **accelerating** and the **velocity** is getting **larger**.
4) A **flattening curve** means the **object** is **decelerating** and the **velocity** is getting **smaller**.







Steeper gradient = greater velocity — except when I try to run up a hill...

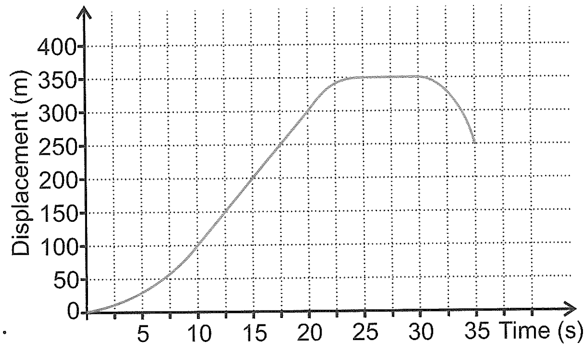
1) Sketch separate displacement-time graphs for a car in each of the following situations:
   a) Travelling away from the observer at a constant velocity.
   b) Travelling away from the observer and slowing down.
   c) Not moving, a short distance from the observer.
   d) Accelerating towards the observer.

Displacement-Time Graphs

EXAMPLE: The displacement-time graph below shows a motorcyclist accelerating to a constant speed, braking and then riding a short distance in the opposite direction.

You can read the following directly off the graph:

1) He took 10 s to accelerate to full speed and he travelled 100 m in that time.
2) He travelled at a constant velocity for the next 10 s and he travelled 200 m in that time.
3) He took 5 s to decelerate (by braking) and stop. He travelled 50 m in that time.
4) He remained stationary for 5 s at a displacement of 350 m from his starting point.
5) He accelerated in the opposite direction for 5 s.



You can work out three more details of the motorcyclist's journey:

1) The value of the **constant velocity** he travelled at between **10** and **20 seconds**.

$$\text{velocity} = \text{gradient} = \frac{\text{change in displacement}}{\text{change in time}} = \frac{300-100}{20-10} = \frac{200}{10} = \mathbf{20\ ms^{-1}}$$

2) His **average velocity** for the **whole journey** — found by dividing his **overall change in displacement** by the **journey time**.

$$\text{average velocity} = \frac{\text{final displacement} - \text{initial displacement}}{\text{total time taken}}$$

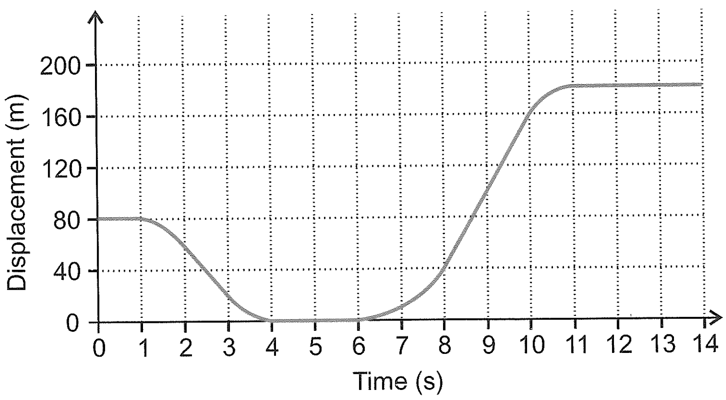
$$= \frac{250-0}{35} = \frac{250}{35} = 7.142... = \mathbf{7.1\ ms^{-1}\ (to\ 2\ s.f.)}$$

3) His **average speed** for the **whole journey** — found by dividing his **total distance travelled** by the **journey time**.
The total distance is the distance travelled in the positive direction (350 m) plus the distance travelled in the negative direction (100 m).

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{350+100}{35} = = 12.85... = \mathbf{13\ ms^{-1}\ (to\ 2\ s.f.)}$$

Displacements — pretty lousy work experience if you ask me...

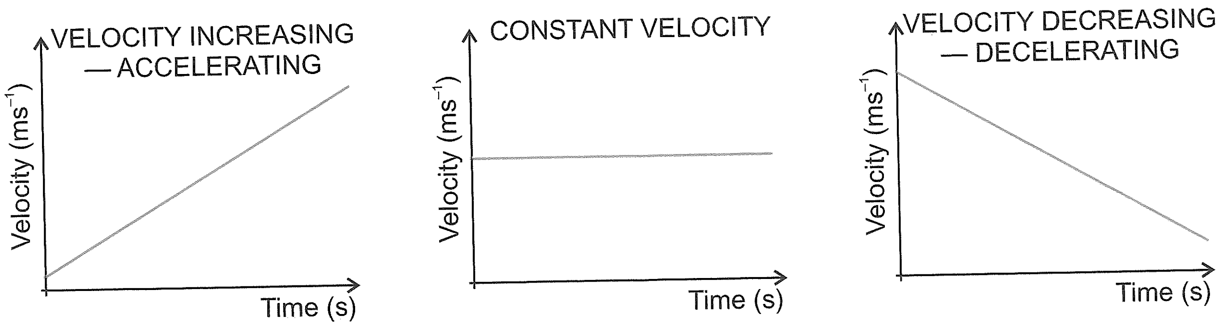
1) The displacement-time graph below shows the displacement of a racing car from the start line.
   a) Is the car accelerating or decelerating between 1 s and 2 s?
   b) Describe the motion of the car between 3 s and 6 s.
   c) What is the velocity of the car between 8 s and 10 s?
   d) What is the car's average velocity for the entire journey?
   e) What is the car's average speed for the entire journey?



Velocity-Time Graphs

You can *Draw Graphs* to Show the *Velocity* of an *Object*

You can also draw graphs that show the **velocity** of an object moving in one dimension.



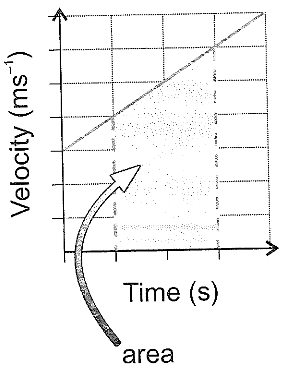
You can use a velocity-time graph to calculate two things:

1) The **distance** the object has moved.
2) The **acceleration**.

The *Area Under the Line* is the *Distance Travelled*

To find the **distance** an object **travels between two times**:

1) Draw **vertical lines** up from the horizontal axis at the two times.
2) Work out the **area** of the shape formed by these lines.
3) When you work out the area, you're **multiplying time** (the horizontal length) by **average speed** (the average vertical length), so the result is a **distance**.
4) You can work out the area in **two ways**:
   - Divide the shape into **trapeziums**, **triangles**, and/or **rectangles** and add up the **area** of each one.
   - Or work out how many metres **each grid square** on the graph is worth, then **multiply by** the **number of squares under the line**. For squares cut by a **diagonal part** of the line, you'll need to **estimate** the **fraction** of the square that's under the line.



EXAMPLE: What is the distance travelled between 1 second and 5 seconds?

The shape made by the area between 1 and 5 seconds can be divided into a rectangle, a trapezium and a triangle.

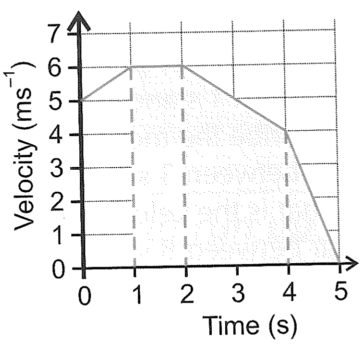
So the total area = area of rectangle + area of trapezium + area of triangle.

Area of rectangle = width × height = 1 × 6 = 6 m

Area of trapezium = ½ × (left side + right side) × width
= ½ × (6 + 4) × 2 = 5 × 2
= 10 m

Area of triangle = ½ × width × height = ½ × 1 × 4 = 2 m

So distance travelled = 6 + 10 + 2 = **18 m**



Velocity-Time Graphs

The Gradient of the Line is the Acceleration

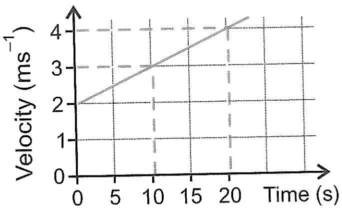
The **acceleration** of an object travelling in **one dimension** (see page 6) is given by:

$$\textbf{Acceleration} \text{ (in ms}^{-2}\text{)} = \frac{\textbf{change in velocity} \text{ (in ms}^{-2}\text{)}}{\textbf{time taken} \text{ (in s)}}$$

This is just the **gradient** of a velocity-time graph. This means that a velocity-time graph of an object's motion has a **negative gradient** when an object is **slowing down** (decelerating).

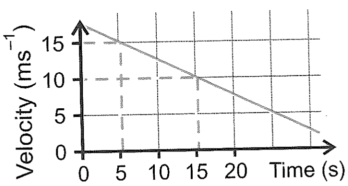
EXAMPLE: What is the acceleration between 10 and 20 seconds?

$$\text{Acceleration} = \frac{4-3}{20-10} = \frac{1}{10} = \mathbf{0.1\ ms^{-2}}$$



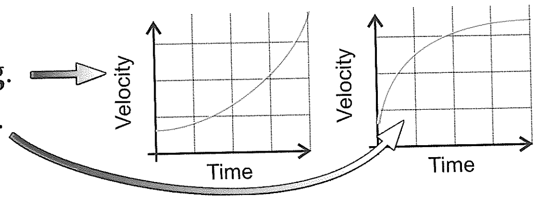
EXAMPLE: What is the acceleration between 5 and 15 seconds?

$$\text{Acceleration} = \frac{10-15}{15-5} = \frac{-5}{10} = \mathbf{-0.5\ ms^{-2}}$$



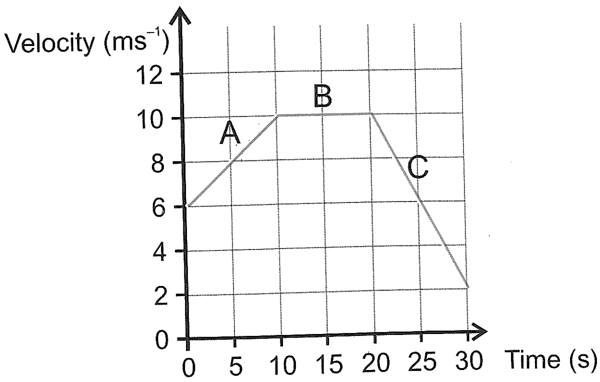
A Curved Line means the Acceleration is Changing

If the line is curved, the acceleration is **not constant**.
A **steepening** curve means the acceleration is **increasing**.
A **flattening** curve means the acceleration is **decreasing**.

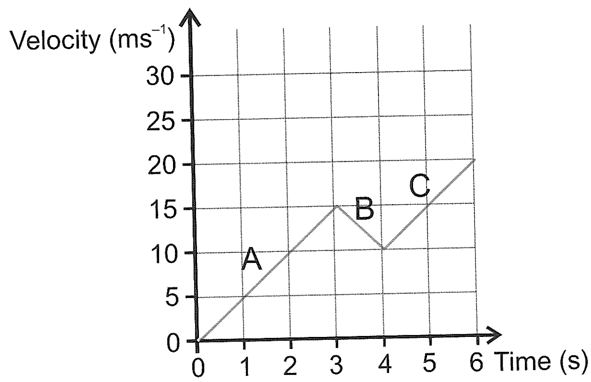


A steepening curve — my v-t graph when I find a spider in my room...

1)



2)

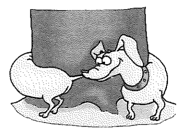


a) Calculate the acceleration shown in sections A, B and C on each graph.
b) Calculate the total distance travelled shown by each graph.

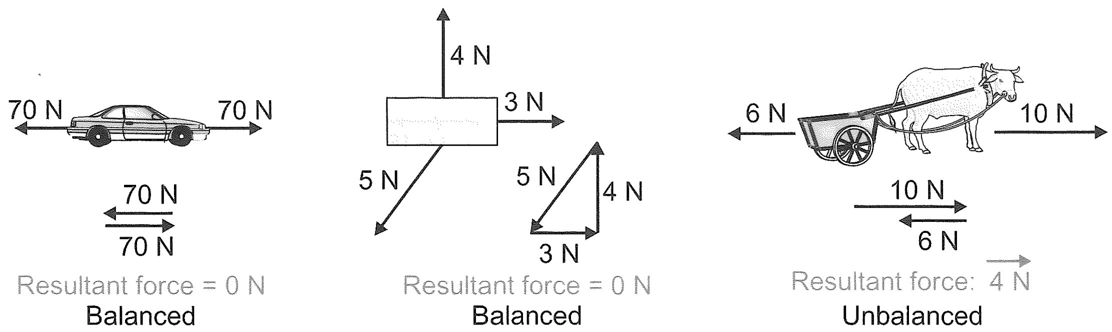
Adding and Resolving Forces

*The **Resultant Force** is the **Sum** of **All** the **Forces***

1) Force is a **vector**, just like displacement or velocity.
2) When **more than one force** acts on a body, you can **add them together** in just the same way as you add displacements or velocities.
3) You find the **resultant force** by putting the arrows "tip-to-tail".
4) If the resultant force is **zero**, the forces are **balanced**.
5) If there's a resultant force, the forces are **unbalanced** and there's a **net force** on the object.



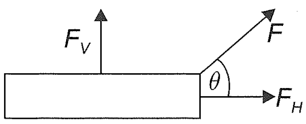
EXAMPLE: Find the resultant force on each object below and decide if the forces are balanced or unbalanced.



*You can **Resolve Forces** just like **Other Vectors***

1) Forces can be in **any direction**, so they're not always at right angles to each other. This is sometimes a bit **awkward** for **calculations**.
2) To make an 'awkward' force easier to deal with, you can think of it as **two separate forces**, acting at **right angles** to **each other**. Forces are **vectors**, so you just use the method on p.5.

The force $\boldsymbol{F}$ has exactly the same effect as the horizontal and vertical forces, $\boldsymbol{F_H}$ and $\boldsymbol{F_V}$. Use these formulas when resolving forces:

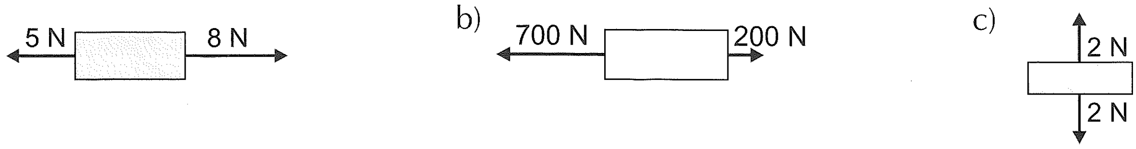


$$F_H = F\cos\theta \text{ and } F_V = F\sin\theta$$

Unbalanced forces — a police officer and a tank on a seesaw...

1) Work out the resultant forces on these objects. Are the forces are balanced or unbalanced?

a) b) c)



2) The engine of a plane provides a force of 920 N at an angle of 12° above the horizontal. What is the horizontal component of the force?
3) A kite surfer is pulled along a beach by a force of 150 N at an angle of 78° above the horizontal. What is the vertical component of the force?

Forces and Acceleration

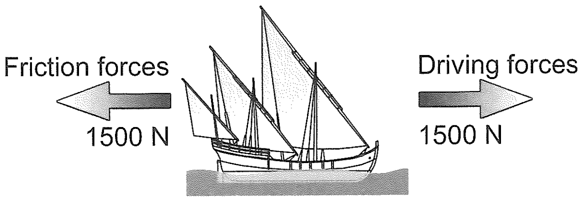
Newton's *First Law* — a *Force* is Needed to *Change Velocity*

1) It's difficult to explain exactly what a "force" is, so instead we talk about what forces do.
2) Forces **stretch**, **squash** or **twist** things, but most importantly forces make things go **faster** (or **slower** or **change direction**).
3) **Newton's First Law** says that:

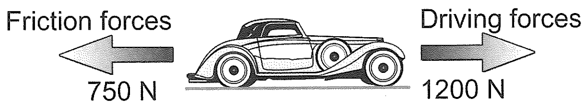
   The velocity of an object **will not change** unless a **resultant force** acts on it.

4) This means an object will **stay still** or **move** in a **straight line** at a **constant speed**, unless there's a **resultant force acting on it**.
5) A **resultant force** is when the forces acting on an object are **unbalanced** (see p.12) — e.g. when a car accelerates, the driving force from the engine is greater than the friction forces.
6) If there's a resultant force, the object will **accelerate** in the **direction** of the resultant force.

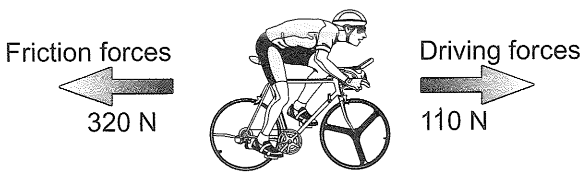
EXAMPLE: How does the velocity change in each of these examples?



Resultant force = 1500 – 1500 = 0 N
No acceleration, so velocity doesn't change.



Resultant force = 1200 – 750 = 450 N
Car accelerates, so velocity increases.



Resultant force = 110 – 320 = –210 N
Bike decelerates, so velocity decreases.

Newton's *Second Law* — *Acceleration* is *Proportional* to *Force*

1) According to Newton's First Law, applying a resultant force to an object makes it accelerate.
2) **Newton's Second Law** says that:

   The **acceleration** is **directly proportional** to the **resultant force**.

3) This means that if you **double** the **force applied** to an object, you **double its acceleration**.
4) You can write down this relationship as the equation:

   resultant force (in newtons, N) = **mass** of object (in kg) × **acceleration** of object (in ms^{-2})

   Or, in symbols:

$$F = m \times a$$

Forces and Acceleration

Here are some **Examples** of Newton's **Second Law**

EXAMPLE: A car of mass 1250 kg accelerates uniformly from rest to 15 ms^{-1} in 25 s. What is the resultant force accelerating it?

$v = 15\ ms^{-1}$, $u = 0\ ms^{-1}$, $t = 25$ s

$a = \frac{v-u}{t}$, so $a = \frac{15-0}{25} = 0.60\ ms^{-2}$

Then $F = m \times a = 1250 \times 0.60 =$ **750 N**

EXAMPLE: A cyclist applies a braking force of 150 N to come to a stop from a speed of 2.5 ms^{-1} in 2.3 s. What is the total mass of the cyclist and their bike?

$v = 0\ ms^{-1}$, $u = 2.5\ ms^{-1}$, $t = 2.3$ s

Again $a = \frac{v-u}{t} = \frac{0-2.5}{2.3} = -1.086...\ ms^{-2}$

The acceleration is negative because the cyclist is slowing down — the acceleration and the resultant force are in the opposite direction to the cyclist's motion.

Then $m = \frac{F}{a} = \frac{-150}{-1.086...} = 138 =$ **140 kg (to 2 s.f.)**

(This answer is rounded to 2 s.f. to match the data in the question — see page 1.)

Newton's **Third Law** — Forces have an **Equal, Opposite Reaction**

Newton's Third Law says that:

Each force has an **equal** and **opposite reaction** force.

This means that if object A exerts a force on object B, then object B must exert an **equal but opposite** force on object A.

For example — when you are **standing up**, you exert a force (your weight) on the floor and the floor **pushes back** with a force of the **same size** in the **opposite direction**.
If it didn't you'd just **fall through the floor**...

Newton was awful at times tables — he was only interested in fours...

1) A car pulls a caravan of mass 840 kg. If the car accelerates at 0.50 ms^{-2}, what force will the caravan experience?
2) An apple of mass 0.120 kg falls with an acceleration of 9.81 ms^{-2}. What is the gravitational force pulling it down (its weight)?
3) An arrow of mass 0.5 kg is shot from a bow. If the force from the bow-string is 250 N, what is the initial acceleration of the arrow?
4) What is the mass of a ship if a force of 55 000 N produces an acceleration of 0.275 ms^{-2}?
5) A train of mass 15 000 kg accelerates from rest for 25 s. If the total force from the engines is 8600 N, what is the train's final velocity?

Kinetic Energy

Moving Things Have *Kinetic Energy*

Energy is a curious thing. You can't pick it up and look at it.
One thing's for certain though — if you're **moving** then you have energy.
This movement energy is more properly known as **kinetic energy**,
and there's a formula for working it out:

If a body of **mass *m*** (in kilograms) is moving with **speed *v*** (in metres per second) then its **kinetic energy** (in joules) is given by:

kinetic energy = ½ × mass × speed²

Or, in symbols: $E_k = \frac{1}{2} \times m \times v^2$

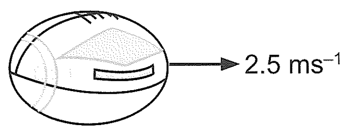
Have a look at the following examples, and then try the questions after them.

EXAMPLE: A car of mass 1000 kg is travelling with a speed of 20 ms^{-1}.
What is its kinetic energy?

$E_k = \frac{1}{2} \times m \times v^2$, so $E_k = \frac{1}{2} \times 1000 \times 20^2$
$= \frac{1}{2} \times 1000 \times 400 = 200\,000 = \mathbf{2 \times 10^5\ J}$

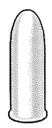
EXAMPLE: A ball has a speed of 2.5 ms^{-1} and has kinetic energy equal to 0.75 J. What is the mass of the ball?

$E_k = \frac{1}{2} \times m \times v^2$
Multiplying both sides by 2 gives $2 \times E_k = m \times v^2$,
then dividing both sides by v^2 gives $\frac{2 \times E_k}{v^2} = m$,
so $m = \frac{2 \times E_k}{v^2} = \frac{2 \times 0.75}{2.5 \times 2.5} = \frac{1.5}{6.25} = \mathbf{0.24\ kg}$



EXAMPLE: A bullet has kinetic energy equal to 1200 J.
If its mass is 15 g, what is its speed?

$m = 15\ g = 0.015\ kg$
From the previous example: $2 \times E_k = m \times v^2$
Dividing both sides by m gives $\frac{2 \times E_k}{m} = v^2$,
then taking square roots of both sides gives $\sqrt{\frac{2 \times E_k}{m}} = v$,
so $v = \sqrt{\frac{2 \times E_k}{m}} = \sqrt{\frac{2 \times 1200}{0.015}} = \mathbf{400\ ms^{-1}}$



Kinetic energy — what you need lots of when you're late for the bus...

1) An arrow of mass 0.125 kg is travelling at a speed of 72.0 ms^{-1}. What is its kinetic energy?
2) A ship has kinetic energy equal to 5.4×10^7 J when moving at 15 ms^{-1}. What is its mass?
3) A snail of mass 57 g has a kinetic energy of 1.0×10^{-6} J. What is its speed?

Gravitational Potential Energy

Gravitational Potential Energy *Depends on* ***Height*** *and* ***Mass***

When an object **falls**, its speed **increases**. As its speed increases, so does its **kinetic energy**. **Where** does it get this energy from?
Answer — from the **gravitational potential energy** it had before it fell:

> If a body of **mass *m*** (in kilograms) is **raised** through a **height *h*** (in metres), the **gravitational potential energy** (in joules) it gains is given by:
> **gravitational potential energy = mass × gravitational field strength × height**

So, in symbols it reads: $\boldsymbol{E_p = m \times g \times h}$

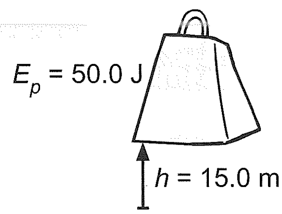
The gravitational field strength, *g*, is the **ratio** of an object's **weight** to its **mass** (in newtons per kilogram, Nkg^{-1}).
At the surface of the Earth, *g* has an approximate value of **9.81 Nkg^{-1}**.

EXAMPLE: An 80.0 kilogram person in a lift is raised 45.0 metres. What is the increase in the person's gravitational potential energy?

$E_p = m \times g \times h$, so $E_p = 80.0 \times 9.81 \times 45.0 = 35\,316 =$ **35 300 J (to 3 s.f.)**

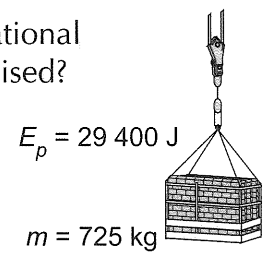
EXAMPLE: A mass raised 15.0 metres gains gravitational potential energy equal to 50.0 joules. What is that mass?

$E_p = m \times g \times h$. Dividing both sides by *g* and *h* gives $\frac{E_p}{g \times h} = m$,
so $m = \frac{E_p}{g \times h} = \frac{50.0}{9.81 \times 15.0} = 0.3397... =$ **0.340 kg (to 3 s.f.)**



EXAMPLE: 725 kilograms of bricks are given 29 400 joules of gravitational potential energy. Through what height have they been raised?

$E_p = m \times g \times h$. Dividing both sides by *m and g* gives $\frac{E_p}{m \times g} = h$,
so $h = \frac{E_p}{m \times g} = \frac{29\,400}{725 \times 9.81} = 4.1337... =$ **4.13 m (to 3 s.f.)**



Liven up your roasts — pour on some graveytational potential energy...

1) How much more gravitational potential energy does a 750 kg car have at the top of a 350 m high hill than at the bottom?
2) A crate is raised through 7.00 metres and gains 1715 J of gravitational potential energy. What is the mass of the crate?
3) A 65.0 kilogram hiker gains 24 700 joules of gravitational potential energy when climbing a small hill. How high have they climbed?

Conservation of Energy

The Conservation of Energy Applies to Falling Bodies

The principle of **conservation of energy** states that:

"Energy **cannot** be **created** or **destroyed** — it can only be **converted** into other forms"

So as long as you ignore air resistance...
...for a **falling** object:

kinetic energy gained (in joules) = **gravitational potential energy lost** (in joules)

...and for an object **thrown** or **catapulted** upwards:

gravitational potential energy gained (in joules) = **kinetic energy lost** (in joules)

This can be very useful in solving problems.
Read through the examples and then have a go at the questions afterwards.
(In all the questions, you can ignore air resistance.)

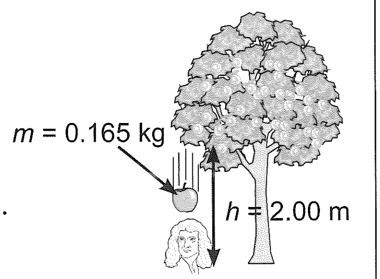
EXAMPLE: An apple of mass 0.165 kilograms falls 2.00 metres from a tree. What speed does it hit the ground at?

E_p lost = $m \times g \times h = 0.165 \times 9.81 \times 2.00 = 3.2373$ J

Therefore E_k gained = 3.2373 J. $E_k = \frac{1}{2} \times m \times v^2$.

Rearranging this gives $v = \sqrt{\frac{2 \times E_k}{m}}$, so $v = \sqrt{\frac{2 \times 3.2373}{0.165}} = 6.264...$

= **6.26 ms⁻¹ (to 3 s.f.)**



EXAMPLE: A model clown of mass 225 grams is fired straight upwards from a cannon at 10.0 metres per second. How high does it get?

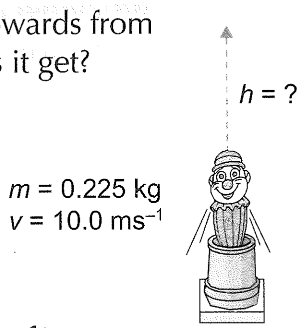
m = 225 g = 0.225 kg

E_k lost = $\frac{1}{2} \times m \times v^2 = \frac{1}{2} \times 0.225 \times 10.0^2 = 11.25$ J

Therefore, E_p gained = 11.25 J. $E_p = m \times g \times h$.

Rearranging this gives $h = \frac{E_p}{m \times g}$, so $h = \frac{11.25}{0.225 \times 9.81}$

= 5.096... = **5.10 m (to 3 s.f.)**



Today I'm practising conservation of energy — I'm staying in bed all day...

1) A gymnast jumps vertically upwards from a trampoline with 2850 J of kinetic energy. They climb to a height of 5.10 m. What is the gymnast's mass?
2) A book of mass 0.475 kilograms falls off a table top 92.0 centimetres from the floor. What speed is it travelling at when it hits the floor?
3) A bullet of mass 0.015 kilograms is fired upwards at 420 ms^{-1}. What height does it reach?

Work

Work — the Amount of Energy Transferred by a Force

When you **move** an object by **applying a force** to it, you are **doing work** (generally against another force) and **transferring energy**. For example:

1) **Lifting** up a box — you are doing work against gravity. The energy is transferred to gravitational potential energy.
2) **Pushing** a wheely chair across a room — you are doing work against friction. The energy is transferred to heat and kinetic energy.
3) **Stretching** a spring — you are doing work against the stiffness of the spring. The energy is transferred to elastic potential energy stored in the spring.

The amount of energy (in joules) that a force transfers is called the **work done**. It's given by:

work done by a force (in joules) = **size of force** (in newtons) × **distance the object moves** in the direction of the force while the force is acting (in metres)

Or, in symbols: $W = F \times s$

EXAMPLE: A 5.0 newton force pushes a box 3.0 metres in the same direction as the force. What is the work done by the force?

$W = F \times s$, so $W = 5.0 \times 3.0 =$ **15 J**

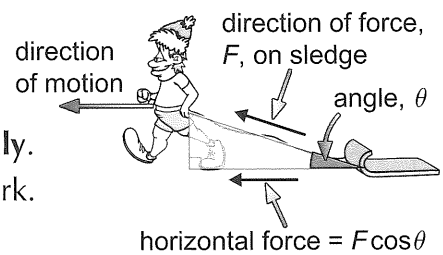
The Force isn't Always in the Same Direction as the Movement

Sometimes the force acts in a **different direction** to the object's movement.

For example — when you **pull** on a sledge, the force acts **diagonally** along the rope but the sledge only moves **horizontally**.

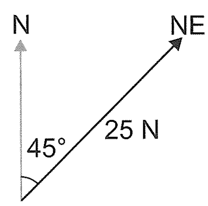
So it's only the **horizontal part** of the force that is doing any work.

You need to use some **trigonometry** to find the work done:



$W = F \cos\theta \times s$ (See page 12 for more about resolving forces.)

EXAMPLE: A 25 newton force to the north-east pushes an object 15 metres in a northerly direction. What is the work done?



Use trigonometry to find the part of the force that acts in the direction of travel (i.e. north).

North-east = 045°, so $F \cos\theta = 25 \times \cos 45° = 17.677...$ N

So the work done is $W = F \cos\theta \times s = 17.677... \times 15 = 265.165... =$ **270 J (to 2 s.f.)**

Work is F times s, what a way to make a living...

1) An upwards force of 25 newtons lifts an object 44 metres. What is the work done?
2) A boy pulls a toy cart 2.5 m along the ground. He applies a force of 17 N at an angle of 35° to the horizontal. How much work does he do?

Work

Work Done = *Increase* in *Gravitational* and *Kinetic Energy*

If a force does work on an object, a few things can happen. For example:

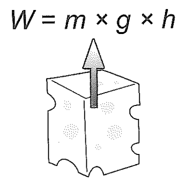
The **work done** can go **entirely** into the **gravitational potential energy** of the object:

EXAMPLE: A force does 74 J of work lifting a 3.0 kg cheese straight up. How high is the cheese lifted?

Work done = increase in gravitational potential energy, so:

$W = m \times g \times h$, and so $h = \frac{W}{m \times g}$

$h = \frac{74}{3.0 \times 9.81} = 2.514... = $ **2.5 m (to 2 s.f.)**



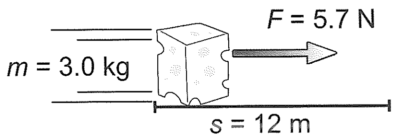
The **work done** can go **entirely** into the **kinetic energy** of the object:

EXAMPLE: The same cheese (of mass 3.0 kg) is pushed horizontally along a frictionless surface with a force of 5.7 N over a distance of 12 m. What is its final speed, assuming it was initially at rest?

Work done = increase in kinetic energy, so:

$W = F \times s = ½ \times m \times v^2$, so $v = \sqrt{2 \times \frac{F \times s}{m}}$

$v = \sqrt{2 \times \frac{5.7 \times 12}{3.0}} = 6.752... = $ **6.8 ms⁻¹ (to 2 s.f.)**



The **work done** can go into increasing **both** the **kinetic** and the **gravitational energy**:

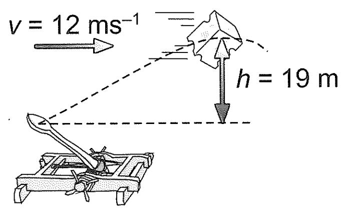
EXAMPLE: The same cheese is fired diagonally upwards from a catapult. At its highest point it has climbed 19 m and is moving horizontally at 12 ms^{-1}. How much work was done on the cheese?

Work done = increase in E_k + increase in E_p, so:

$F \times s = (½ \times m \times v^2) + (m \times g \times h)$

$= (½ \times 3.0 \times 12^2) + (3.0 \times 9.81 \times 19.0)$

$= 775.17 = $ **780 J (to 2 s.f.)**



Work done? No, you need to answer this question first...

1) A constant 125 N force lifts a 5.75 kg rocket vertically upwards. When the rocket reaches a height of 2.50 m the force is removed, but the rocket continues to move upwards. Calculate:
   a) the work done by the force.
   b) the gain in gravitational potential energy.
   c) the gain in kinetic energy.
   d) the upwards speed of the rocket immediately after the force is removed.

Power

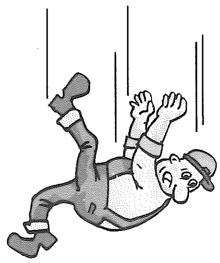
Power — the Work Done Every Second

In mechanical situations, **whenever** energy is **converted**, **work** is being done.

For example, when an object is **falling**, the force of **gravity** is doing work on that object **equal** to the **increase** in **kinetic energy** (ignoring air resistance).

The **rate** at which this work is being done is called the **power**.

You can calculate it using:



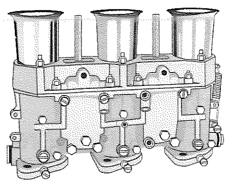
power (in watts) = **work done** (in joules) ÷ **time taken** (in seconds)

Or, in symbols: $P = \frac{W}{t}$

Power is measured in **watts**.

A watt is equivalent to **one joule of work done per second**.

> **EXAMPLE:** If 10 joules of work are done in 2 seconds, what is the power?
>
> $P = W \div t = 10 \div 2 = \mathbf{5\ W}$

> **EXAMPLE:** For how long must a 3.2 kilowatt (3.2×10^3 watt) engine run to do 480 kilojoules (4.8×10^5 joules) of work?
>
> $P = W \div t$
>
> Multiplying both sides by t gives: $P \times t = W$
>
> Then dividing both sides by P gives: $t = W \div P$
>
> So, $t = W \div P = \frac{4.8 \times 10^5}{3.2 \times 10^3} = \mathbf{150\ s}$
>
> 

> **EXAMPLE:** A force of 125 newtons pushes a crate 5.2 metres in 2.6 seconds. What is the power? (The motion is in the same direction as the force.)
>
> First you need to find the work done (see page 18):
>
> $W = F \times s = 125 \times 5.2 = 650$ J
>
> Then use W to find the power:
>
> $P = W \div t = 650 \div 2.6 = \mathbf{250\ W}$

The power of love ain't that special — it's just a lot of work over time...

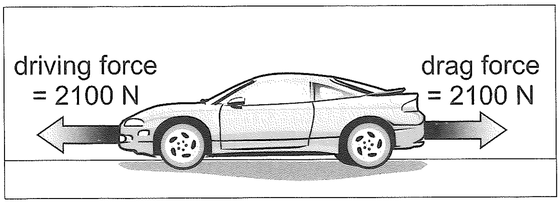
1) What is the power output of a motor if it does 250 joules of work in 4.0 seconds?
2) If a lift mechanism works at 14 kilowatts, how long does it take to do 91 kilojoules of work?
3) An engine provides a force of 276 N to push an object 1.25 km in 2.5 minutes. What power is the engine working at?

Power

Power *is also* ***Force*** *Multiplied By* ***Speed***

There's a **useful equation** you can **derive** for the **work done** by a force **every second** on an object moving at a **constant speed**. Follow through the working in the example below:

EXAMPLE: What power is a car engine working at if it produces a driving force of 2100 newtons when moving at a steady speed of 32 metres per second?



The car is moving at a steady speed. This means the forces on it are balanced, so the driving force must be equal to the drag force.

The power of the engine is given by $P = W \div t$.

$W = F \times s$, so we can substitute for the work done, giving $P = \frac{F \times s}{t}$.

Now, $\frac{F \times s}{t}$ is the same as $F \times \frac{s}{t}$, so $P = F \times \frac{s}{t}$.

Finally we use the fact that $\frac{s}{t} = \frac{\text{distance travelled}}{\text{time taken}}$ = the speed, v.

$$\text{So, } \boldsymbol{P = F \times \frac{s}{t} = F \times v}$$

power (in watts) = **force** (in newtons) × **speed** (in metres per second)

For our example, $P = 2100 \times 32 = 67\,200 =$ **67 000 W** (or 67 kW) **(to 2 s.f.)**

(This answer is rounded to 2 s.f. to match the data in the question — see page 1.)

IMPORTANT:
The formula $P = F \times v$ is **only** true when the object is moving at a **constant speed** in the **same direction as the force**.

Mooving forces with a lot of power — a stampeding herd of cows...

1) What is the power delivered by a train engine if its driving force of 1.80×10^5 newtons produces a constant speed of 40.0 metres per second?
2) A skydiver is falling at a constant velocity of 45 metres per second. Gravity is doing work on her at a rate of 31 500 joules per second. What is her weight?
3) A car is travelling at steady speed. Its engine delivers a power of 5.20×10^4 watts to provide a force of 1650 newtons. What speed is the car travelling at (in metres per second)?

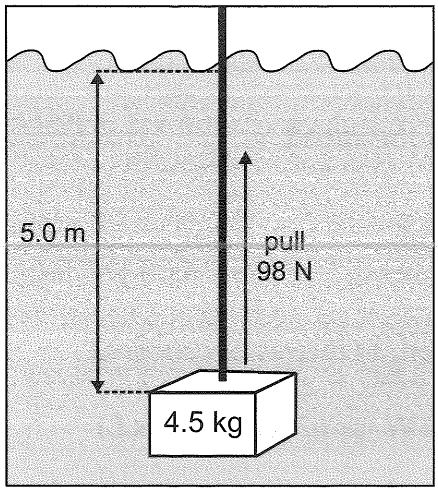
Efficiency

How Much of What You **Put In** Do You **Get Out**?

1) For most mechanical systems you **put in** energy in **one form** and the system **gives out** energy in **another**.
2) However, **some** energy is **always** converted into forms that **aren't useful**.
3) For example, an electric motor converts electrical energy into **heat** and **sound** as well as useful kinetic energy.
4) You can measure the **efficiency** of a system by the **percentage of total energy put in that is converted to useful forms**.

$$\text{Efficiency} = \frac{\text{Useful energy out}}{\text{Total energy in}} \times 100\%$$

EXAMPLE: A pirate uses a rope to pull a box of mass 4.5 kg vertically upwards through 5.0 m of water. He pulls with a force of 98 N. What is the efficiency of this system?



The **energy the pirate puts in** is the work he does pulling the rope.

The **useful energy out** is the gravitational potential energy gained by the box.

Some energy is converted to heat and sound by **friction** as the box is dragged through the water.

Total energy in = work done = $F \times s$
= 98×5.0
= 490 J

Useful energy out = gravitational potential energy gained
= $m \times g \times h$
= $4.5 \times 9.81 \times 5.0$
= 220.725 J

So, efficiency $= \frac{\text{Useful energy out}}{\text{Total energy in}} \times 100\%$

$= \frac{220.725}{490} \times 100\% = 45.045... =$ **45% (to 2 s.f.)**

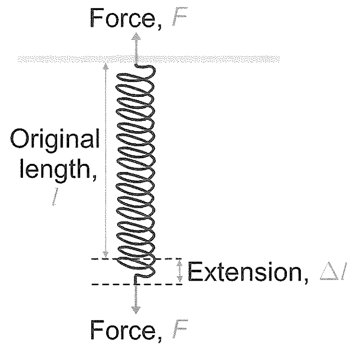
Efficiency – getting on with these questions instead of messing about...

1) A motor uses 375 joules of electrical energy in lifting a 12.9 kilogram mass through 2.50 metres. What is its efficiency?
2) It takes 1.4 megajoules (1.4×10^6 joules) of chemical energy from the petrol in a car engine to accelerate a 560 kilogram car from rest to 25 metres per second on a flat road.
   a) What is the gain in kinetic energy?
   b) What is the efficiency of the car?

Forces and Springs

Hooke's Law — Extension is Directly Proportional to Force

1) When you apply a **force** to an object you can cause it to **stretch** and **deform** (change shape).
2) **Elastic objects** are objects that return to their **original shape** after this deforming force is **removed**, e.g. springs.
3) When a **spring** is supported at the top and a **weight** is attached to the bottom, it **stretches**.
4) The **extension**, Δl, of a spring is **directly proportional** to the **force** applied, F. This is **Hooke's Law**.
5) This relationship is also true for many other elastic objects like **metal wires**.



force (in newtons, N) = **spring constant** (in newtons per metre, Nm^{-1}) × **extension** (in metres, m)

$$F = k \times \Delta l$$

The **spring constant**, k, depends on the stiffness of the **material** that you are stretching. It's measured in **newtons per metre** (Nm^{-1}).

EXAMPLE: A force is applied to a spring with a spring constant of 65.0 Nm^{-1}. The spring extends by 12.3 cm. What size is the force?

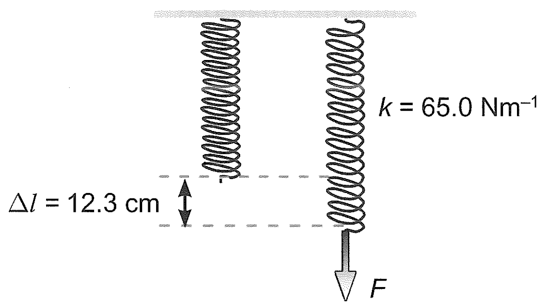
$F = k \times \Delta l$

Δl = 12.3 cm = 0.123 m

So, $F = 65.0 \times 0.123$

= 7.995

= **8.00 N (to 3 s.f.)**



EXAMPLE: A sack of flour of mass 7.10 kg is attached to the bottom of a vertical spring. The spring constant is 85.0 Nm^{-1} and the top of the spring is supported. How much does the spring extend by?

$F = k \times \Delta l$, so $\Delta l = \frac{F}{k}$

You need to work out the force from the given mass:

F = weight of flour = $m \times g$

= 7.10 × 9.81 = 69.651 N

So, $\Delta l = \frac{69.651}{85.0}$

= 0.8194...

= **0.819 m (to 3 s.f.)**